Algorithms for 3-D Geometric Bin Packing

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1 Introduction

The bin packing problem has been the corner stone of approximation algorithms and has been extensively studied [GJ81, Lod99] starting from the classical work of Garey and Johnson [GJ79]. The problem is also important from a practical standpoint and finds applications in scheduling and routing. In this proposal we will present variant of this problem that is motivated by its applications in *palletizing* problems.

Concretely, we will study the 3D bin-packing problem where we are given a set of rectangular items $A_1, A_2 \cdots A_n \in (0,1]^3$ specified by their depth, width and height and the goal is to pack these items into a minimum number of unit cube bins. Apart from the packing constraints we would also like to incorporate additional considerations such as stability, center of gravity, elasticity, and shapes of items.

2 Problem Statement and Prior Work

3D Bin Packing: In the 3-Dimensional (geometric) Bin Packing problem, we are given a set of 3-Dimensional rectangular items whose size along each dimension is bounded by one, and the goal is to pack these items into a minimum number of unit cube bins. We will consider *orthogonal packing without rotation* i.e. the items are not allowed to be rotated and must be packed parallel to the edges of the box. In any feasible solution, items are not allowed to overlap.

In this proposal we also intend to study solutions to the 3-D Bin Packing problem, which are realizable for example while stacking items for shipping or storage. We will focus on three aspects of this problem:

1. **Gravity:** We wish to ensure that our solution is stable under gravity i.e. there are no overhanging or floating boxes. Almost all known algorithms for bin packing use stage packing or level-oriented packing [Cap02] where items are first packed into shelves that are then packed into bins. In such solutions the bottom of one item in a shelf might not be touching the top of the item just below in the lower shelf and may lead to unstable solutions under gravity.

2. **Unpacking Sequence:** Sometimes we need to consider the unloading sequence while doing the bin packing. For example, when UPS fills up their delivery trucks they have to pack items in minimum number of trucks in such a way that they can unload it in the order the delivery places arrive. In other words, we are given with a total order and set of items to be packed, we need to pack them into minimum number of bins so that if item i appears before item j in the total order, then item j is placed after placing item i. This problem is an easier version of online bin packing as we already know all items that are needed to be packed.

3. **Stability:** We also intend to study stable palletization, where stability is measured by the amount of interlockings (crossing edges between adjacent levels).

The above problems are NP-hard and we will be interested in polynomial time approximation algorithms for the same. Often for bin packing problems it is possible to construct *small* pathological instances where no algorithm attains a reasonable approximation ratio and thus we will use the notion of *asymptotic approximation ratio* (AAR, denoted by R_A^{∞}) as a measure of our performance. Given a poly-time algorithm A, the ratio R_A^{∞} is given by $R_A^{\infty} = lim_{n\to\infty}\ sup\ R_A^n$, where

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$R_A^n = max\{A(I)/OPT(I)|OPT(I) = n\}$ and I ranges over set of all problem instances. A problem is said to admit an *Asymptotic Polynomial Time Approximation Schemes* (APTAS) if for every $\epsilon > 0$, there is a poly-time algorithm with asymptotic approximation ratio $(1 + \epsilon)$.

Prior Work: For 1-D Bin Packing an APTAS is known due to Fernandez de la Vega and Lueker [dlVL81] and Karmarkar and Karp [KK82]. However, Bansal et. al. [BCKS06] showed 2-D Bin Packing in general does not admit an APTAS unless $P = NP$. Caprara in his break-through paper [Cap02] gave an algorithm for 2-D Bin packing attaining an AAR of ≈ 1.69 . This was later improved by Bansal et al. [BCS09] to $(\ln 1.69 + 1) \approx 1.52$ for both the cases with and without rotation. Online version of bin packing is also well-studied. The present best online 1-D, 2-D and 3-D bin packing algorithms have AAR of 1.589[Sei02], 2.55 and 4.31[HCT+11] respectively.

A closely related problem is the *Strip Packing* where we are given 3-D rectangular items each of whose dimensions is at most one and they need to be packed into a single 3-D box of unit depth, unit width and unlimited height so as to minimize the height of the packing. The two dimensional variant of this problem is known as the *Cutting Stock Problem* and is defined analogously. Recently APTAS are given for 2-D Strip Packing without rotations [KR00] and with rotations in [JvS05]. 3-D Strip Packing is a common generalization of both the 2-D Bin Packing problem(when each item has height exactly one) and the 2-D Strip Packing Problem(when each item has width exactly one) and the best known algorithm is by Bansal et al. [BHI+07] with an AAR of 1.69.

3 Solution Proposal or Approach

We propose two approaches to address this problem.
Subset Oblivious Packing: A ρ-approximation algorithm is *subset-oblivious* if it not only produces a solution with value at most ρ *opt*(I) on instance I, but also, given a "random" subset $S \subseteq I$ where each element in S occurs with probability $1/k$, the value of the solution produced by the algorithm on S is bounded by approximately ρ *opt*$(I)/k$. This notion was introduced by Bansal et al. in [BCS09] where they showed that any subset oblivious ρ-approximation algorithm for a d-dimensional bin packing problem can be converted to another randomized algorithm with approximation guarantee close to $\ln \rho + 1$. They found a subset oblivious algorithm for 2-D Bin packing. However they were not able to find a subset-oblivious algorithm for 3-D Bin Packing. The key bottleneck in extending this result to 3D is to find a good approximation algorithm to solve the following LP relaxation of a related set cover problem, in which a set I of items has to be covered by *configurations* from the collection $\mathcal{C} \subseteq 2^I$, where each configuration $C \in \mathcal{C}$ corresponds to a set of items that can be packed into a bin:

$$min\{\sum_{C\in\mathcal{C}} x_C : \sum_{C\ni i} x_C \geq 1 (i \in I), x_C \in \{0,1\} (C \in \mathcal{C})\}\,.$$

The existence of a poly-time algorithm for the above LP relaxation with AAR *poly*(d) would lead to a poly-time algorithm with AAR *poly*(d) for d-D Bin Packing, a significant improvement over current guarantees, which are exponential in d.
Strip Packing: One can easily find a $(1.69 \times 2 + \epsilon) \approx (3.38 + \epsilon)$- asymptotic approximation algorithm for the 3-D Bin packing using the 3-D Strip Packing algorithm in [BHI+07] by cutting the 3-D strip into unit cubes by planes parallel to base and pack the cut items for each plane in a separate bin. *Tall not sliced* property of an algorithm ensures any item that is cut by a plane parallel to the base has height at most ϵ. We will try to show that *tall not sliced property* holds for above algorithm for 3-D Bin Packing when items are Harmonic rounded i.e. each dimension with value greater than some constant ϵ is of the form $1/m$ for some positive integer m. This will give a $(1.69^2 + \epsilon) \approx (2.85 + \epsilon)$ algorithm. We believe the harmonic rounding technique introduced in [LL85] and the analysis of the Next Fit Decreasing Height algorithms[JGJT80] might be useful in this context.

Due to the practical nature of the problems addressed in this proposal we would also be implementing our solutions using commercially available 3D modeling software. Our preliminary experiments indicate that several simple algorithms perform quite well on real-world instances and through this project we also aim to build sufficient theoretical understanding to explain this behavior.

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